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Triaxial Testing in the Design of Shallow Foundations

Essais triaxiale pour la determination de la capacité portante des fondations

The calculation of the ultimate bearing capacity of a shallow foundation is based on the well known Terzaghi formula, which uses superposition to combine the influence of soil weight, surface load, and cohesion. Experimental and theoretical efforts have made this formula one of the best studied and widest used in geotechnical practice. In the last few years our basic knowledge has increased very much, i.e. the use of model laws, the influence of the stress level and the state of stress, the use of flow rules in the theory of plasticity and the principle of superposition. This paper attemps to take that into consideration in the simple case of a vertically, centrally loaded, shallow foundation with a dominating surface load.

INTRODUCTION

The ultimate bearing capacity b of a vertically and centrally loaded, shallow strip footing is normally expressed by

$$b = \frac{1}{2}\gamma B N_\gamma + qN_q + c'N_c \qquad (1)$$

which is known as the Terzaghi equation.

N_γ, N_q and N_c are bearing capacity factors and depend on the effective friction angle ϕ'. γ is the unit weight of the soil, B is the width of the footing, q is the effective overburden pressure at base level and c' is the effective cohesion of the soil.

The formula can be simplified in the undrained case, but this will not be considered here.

The formula consists of three parts, a γ-term, a q-term, and a c'-term. In the case of weightless soil the solution can be obtained by adding $c'\cot\phi'$ to the load at the surface, and after the calculation subtracting $c'\cot\phi'$ from the calculated bearing capacity. The formula can thus be written as:

$$b - c'\cot\phi' = \frac{1}{2}\gamma B N_\gamma + (q + c'\cot\phi')N_q \qquad (2)$$

By neglecting the unit weight of the soil Prandtl (1920) calculated N_q (and N_c) by a statically determined method. The γ-term was calculated in the special case ($q = c' = 0$) by Lundgren and Mortensen (1953). It has been shown that these solutions normally satisfy all conditions of the theory of plasticity. Bent Hansen (1975) has shown that the superposition is not strictly correct.

Brinch Hansen (1961) generalized the formula by adding depth factors, shape factors and inclination factors. For a vertically loaded foundation the formula can be expressed as:

$$b - c'\cot\phi' = \frac{1}{2}\gamma B N_\gamma s_\gamma + (q + c'\cot\phi')N_q s_q d_q \qquad (3)$$

The depth factor d_q can be calculated by means of the theory of plasticity or it can be derived from model tests. The shape factors s_γ and s_q must be found by experiments since it is a 3-dimensional problem. Extensive plate loading tests have been carried out in the laboratory by Meyerhof (1951) and de Beer (1970); tests have also been carried out in the field.

This paper deals with the same problem but attemps to take into consideration the influence of the stress level as measured in triaxial tests. The envelope to the Mohr's circles appears to be curved, but it is possible to describe the envelope by adding a third parameter m to Coulomb's failure condition. The model tests are performed at such high surface loads that the unit weight of the soil can be neglected and most of the laws of similitude can be satisfied. The bearing capacities of small scale foundations can be calculated directly on the basis of the extended Coulomb failure condition by using Prandtl's method. By comparing the theoretical results with model tests it is possible to discuss the influence of the state of stress and the plastic behaviour of the soil on the bearing capacity.

PROPERTIES OF THE SOIL

Two different types of soils have been used with main characteristics as shown in Table I.

Kratbjerg moraine clay

Kratbjerg moraine clay contains 15-20% clay fraction, 25-30% silt fraction and 50-55% sand fraction. It can be characterized as an unweathered, silty moraine clay containing a rather fine sand. It is preconsolidated and has an undrained shear strength c_u = 200 kPa. The geotechnical properties of this soil have been mentioned in details earlier - Moust Jacobsen (1970). In spite of the very low content of clay the soil normally behaves undrained. The tests reported in this paper however, were carried out as drained tests.

Table I Soil characteristics

	Moraine Clay (Kratbjerg)	Sand (Blokhus)
Void ratio e	0.32	0.54-0.59
Water content w	0.12	< 0.001
Liquid limit w_L	0.179	
Plastic limit w_P	0.108	-
Minimum e		0.53
Maximum e		0.79
Mean grain size d_{50}	0.05 mm	0.17 mm
Degree of uniformity $\frac{d_{60}}{d_{10}}$	100	1.5

<u>Blokhus sand</u>

Blokhus sand is a fine, uniform sand from the beach at Blokhus in Jylland. Very dense deposits, $I_D \sim 1$, of dry sand were used. A mass sand spreader has been developed, which uses a pluvial deposition technique. The sand falls from a silo down on a diffuser, which produces a sand rain evenly distributed over the depositing area. In order to obtain a very dense sand the deposition rate has to be small. A 50 cm thick sand deposit is produced in 10 minutes. The variation in dry density was less than 1%. Model deposits as well as triaxial specimens were made in that way.

EXPERIMENTS

<u>Field tests</u> have been carried out on moraine clay. The diameters of the plates were 5 and 15 cm. The test area was as level as possible and had a diameter of at least seven times the diameter of the plate. A thin layer of gypsum was cast between the plate and the soil surface. The tests were performed as controlled strain tests with a strain rate of 0.0005 mm/sec which corresponds to a drained state. The test area was situated above the ground water level, which means that the capillary forces had to be measured. The tests were carried out after an interval of time following excavation so that the overburden pressure, including the capillary forces, was acting as a surface load. The effective cohesion of the soil and the surface load make the q-c term in the formula (3) so important, that the γ-term can be neglected.

<u>Model tests</u> with small plates were performed on dry sand in the laboratory. Two kinds of footings were used, square footings with B = 5 cm and rectangular footings with B x L = 3.5 x 10 cm. The base of the model plate was covered with sand grains to make it perfectly rough. In order to reduce the influence of the unit weight of the soil most of the tests were performed with the surface loads varying between 2.5 and 70 kPa.

The test box is 70 x 70 x 50 cm as shown in Fig. 1. A rubber membrane covered the surface of the sand and the test box. The surface load was applied by means of reduced air pressure within the pores in a similar way as it was done in the field tests on moraine clay. Inside the box strengthening beams prevented lateral deforma-

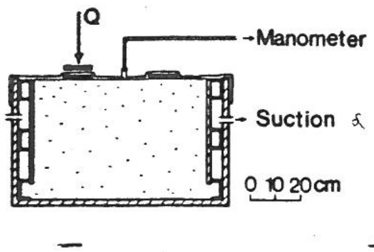

Fig. 1: The test box.

tions caused by suction. The initial state of strain therefore corresponded to that found in nature. Two rectangular plate tests were carried out on the same sand deposit, but it was controlled that the results were not influenced by the walls of the test box. The square plates were placed at the centre of the surface of the deposit.

<u>In triaxial tests</u> smooth pressure heads were used in order to minimize the shear stress at the ends of the specimen. The pressure heads consist of glass plates covered with silicone grease and thin rubber membranes as proposed by Rowe and Barden (1964). When testing clays it was only necessary to use one membrane because the soil surface became rather smooth during preparation. When testing sand samples it was necessary to prevent the sand grains from penetrating the rubber membrane. Therefore a sandwich of four membranes was used. Normally the height of the sample equaled the diameter in order to ensure uniform deformation conditions at failure. The measurements of volume change and vertical deformation thus permit determination of the relative deformations. In a dense specimen with height twice the diameter a distinct rupture face is formed and the measured volume change is of course smaller than in tests with uniform deformation conditions. Further information on the triaxial apparatus has been published previously, Moust Jacobsen (1970).

Normally the size of the cylindrical specimens were 7ø x 7 cm. Some tests though were carried out with 7ø x 14 cm moraine clay and 20ø x 20 cm or 20ø x 40 cm on sand specimens.

ANALYSIS OF TRIAXIAL TEST RESULTS

<u>Dense sand</u>

The results of 51 triaxial tests carried out with Blokhus sand are shown in a (τ_r, σ_3) diagram in Fig. 2. The small circles represent tests with 7ø x 7 cm specimens and the big circles tests with 20ø x 20 cm and 20ø x 40 cm specimens. The small specimens give higher values of τ_r than the big specimens but the deviation is small.

<u>Strength parameters</u>. Analysis of triaxial test results is usually based on the Coulomb failure criterion which may be written:

$$\tau_r = \frac{\sigma_1' - \sigma_3'}{2} = \frac{\sin\phi'}{1 - \sin\phi'}(\sigma_3' + c'\cot\phi') \qquad (4)$$

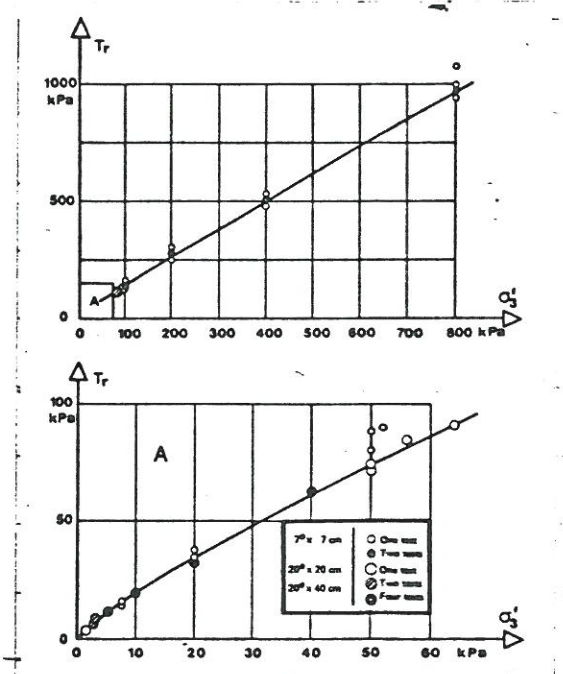

Fig. 2: 51 triaxial tests on dense Blokhus sand. The curved line corresponds to eq (5) with ϕ_a = 32°7 c_a = 11 kPa and m = 0.25.

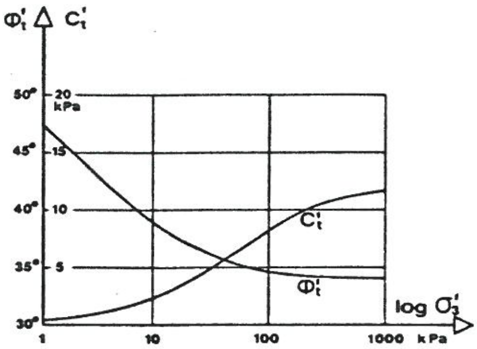

Fig. 3: The tangent friction angle ϕ_t' and tangent cohesion c_t' versus the confining pressure σ_3'.

Results show that for higher stress levels (σ_3' > 100 kPa) the effective strength parameters c' and ϕ' describe the failure state very well.

For lower values of σ_3' the failure curve bends away from the straight line towards origo. Tests at lower stress levels therefore result in higher values of the friction angle ϕ' and lower values of the cohesion c'. This is of very great importance for theoretical considerations based on model tests because the stress level is normally very low in such tests.

The influence of the stress levels can be taken into account by modifying the Coulomb failure condition to:

$$\tau_r = \frac{\sin\phi_a}{1-\sin\phi_a}\,\sigma_3'\left(1+\frac{c_a\cot\phi_a}{m\sigma_3'}\right)^m \qquad (5)$$

$$0 \le m \le 1$$

ϕ_a and c_a are the asymptotic strength parametres at high stress level. For perfect friction soil (c' = c_a = 0) eq (4) and eq (5) are identical, regardless of m. For perfect cohesive soil m = 1 and the two equations become identical for c' = c_a.

There is a very close agreement between eq (5) and the test results, (see Fig. 2).

Another set of strength properties which should be used in the calculation of the bearing capacity is the "tangent" friction angle ϕ_t' and the corresponding cohesion c_t'. According to formula (5) ϕ_t' and c_t' depend on σ_3' as shown in Fig. 3.

Volume expansion. At failure the so-called angle of dilatation can be found from

$$\sin\nu = \frac{\dot\varepsilon_v}{\dot\varepsilon_v - 2\dot\varepsilon_1} \qquad (6)$$

where $\dot\varepsilon_v$ and $\dot\varepsilon_1$ are the rates of relative volume change and relative vertical compression, respectively. In triaxial tests for sand ν is determined to 10° - 15°.

Moraine clay

Drained triaxial tests on moraine clay can be treated in the same way. On Kratbjerg moraine clay specimens ϕ_a = 32°6, c_a = 45 kPa, m = 0.8, and ν = 25° were measured.

Progressive rupture

Dense sand and preconsolidated clay expand at failure. If the strain or stress distribution is not uniform, the first rupture occurs locally, weakening the soil and thus causing the rupture to spread progressively. The measured failure load will thus be smaller than calculated on the basis of an assumed uniform strain distribution.

In the triaxial apparatus a uniform deformation of the sample can be ensured by using specimens whose height equals the diameter and by using smooth pressure heads as shown in Fig. 4. In such tests progressive failure cannot take place.

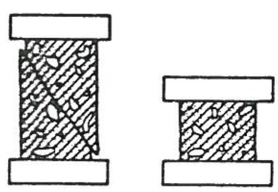

Fig. 4: Specimens with different height - diameter - ratio after failure.

In the soil beneath footings or in small scale model tests progressive rupture can happen and thus might reduce the ultimate load. Therefore it is of great interest to investigate this problem.

In triaxial tests the influence of progressive rupture can be examined by comparing tests performed with specimens of different heights. By giving the specimen a height twice the diameter a single shear plane is formed and the progressive rupture develops.

14 triaxial tests were performed with sand specimens of 40 cm height and diameter 20 cm. 15 triaxial tests were carried out with the same sand at the same void ratio with specimens of height and diameter 20 cm. The results are shown in Fig. 2 as big circles. A closer investigation shows, that the deviation between results from the two groups is not significant. It is thus concluded, that progressive rupture does not influence results of triaxial tests. It might however, influence on the results of plate tests.

A similar test series on moraine clay contains 8 tests. The results are reported elsewhere, Moust Jacobsen (1970) and exhibit a somewhat larger scatter than the sand tests and are thus less conclusive. Progressive rupture might have occurred, but the influence on the failure load is less than 10%.

STATE OF STRESS

Coulombs failure criterion (4), even in the extended form (5), implies that the intermediate principal stress σ_2 has no influence on the shear stress τ_r at failure. But tests in plane apparatus and true triaxial apparatus show, that ϕ' depends on σ_2. In the Danish code of practice for foundation engineering 1965 it was suggested that in the plane state the angle of friction ϕ'_{pl} exceed ϕ'_t by 10 per cent. For a cohesive soil a convenient assumption is, that $c'\cot\phi'$ is independent of the state of stress. Thus

$$\phi'_{pl} = 1{,}1\cdot\phi'_t \tag{8}$$

$$c'_{pl}\cot\phi'_{pl} = c'_t\cdot\cot\phi'_t \tag{9}$$

The bearing capacity of all shapes of footings, even circular, should be calculated by means of ϕ'_{pl}, c'_{pl} because the factors s_γ and s_q are defined as the ratio between the actual bearing capacity and that of a strip footing.

THE USE OF STATICALLY ADMISSIBLE SOLUTIONS

Failure problems are calculated by means of the theory of plasticity. The material should be perfect plastic, which means that the stresses at failure do not depend on the magnitude of the strains and that an associateed flow rule exists for the material. If a correct solution cannot be achieved the upper and lower bound theorems give a maximum and minimum value respectively of the failure load. In this paper some important factors concerning the lower bound theorem will be discussed briefly.

Perfect plastic materials

The bearing capacity Q for strip footings with heavy surface load has been calculated by Prandtl (1920). Overall equilibrium of stresses and external forces is assumed and the body forces are neglected ($\gamma = 0$). Failure is nowhere surpassed. This solution is called statically determined in the theory of plasticity.

A mathematically correct solution to a failure problem in weightless soil must also satisfy the kinematical conditions. The set of strain rates $\dot{\varepsilon}_{ij}$ corresponding to the set of stresses σ_{ij} must be compatible with the displacements rates $\dot{u}_i$ of the surface where the external forces T_i act. If $\dot{\varepsilon}_{ij} \neq 0$, then σ_{ij} fulfils the yield criterion.

The principle of virtual work then gives

$$\int_A T_i\dot{u}_i dA = \int_V \sigma_{ij}\dot{\varepsilon}_{ij} dV \tag{10}$$

The limit theorems are based on the principle of maximum local energy dissipation due to v. Mises

$$\sigma_{ij}\dot{\varepsilon}_{ij} = \text{maximum} \tag{11}$$

which in combination with the yield criterion result in the requirement of orthogonality of the stress and strain rate vectors (the normalty condition or the associated flow rule).

In a statically admissible solution a set of stresses σ^*_{ij} is estimated in such a way that the surface loads T^*_i are balanced and the yield criterion nowhere surpassed. These stresses may not correspond to the strain rates $\dot{\varepsilon}_{ij}$ or the displacement rates $\dot{u}_i$. From eq (10) and (11) it follows that

$$\int_A T^*_i \dot{u}_i dA = \int_V \sigma^*_{ij}\dot{\varepsilon}_{ij} dV \le \int_V \sigma_{ij}\dot{\varepsilon}_{ij} dV$$

It is seen that T^*, which includes the bearing capacity Q^* and the surface load q, determined from a distribution of stresses alone, is less than or equal to T. Thus Q^* is less than or equal to Q of the mathematically correct solution. This is the lower bound solution.

Prandtl's solution is statically admissible, but has been proved to be kinematically admissible also.

Friction materials

For a frictional soil the friction angle ϕ'_t and the angle ν of dilatancy have been defined earlier. For $\nu = \phi$ it can be shown that the material satisfies the normality condition, but the test results show that $\nu < \phi$ i.e., normality does not apply. Hence $\sigma_{ij}\cdot\dot{\varepsilon}_{ij}$ does not take the maximum value for a friction material and the actual bearing capacity must consequently be less than or equal to the bearing capacity for a material satisfying the normality condition. Thus Prandtl's solution must be an upper bound for a friction material. This solution is obtained by setting $\nu = \phi' >$ actual ν.

A lower bound solution can easily be found as Prandtl's solution for $\phi' = \nu <$ the actual ϕ'. But these lower and upper bound solutions will never coincide.

A correct Prandtl solution must therefore correspond to a value $\nu_d = \phi_d'$, where actual $\nu < \nu_d = \phi_d' <$ actual ϕ. ϕ_d' must be found from model tests. The corresponding cohesion c_d' must be found by an additional assumption. A relationship similar to eq (9) is suggested: i.e., $c_d' \cot \phi_d' = c' \cot \phi'$.

Influence of stress level

The influence of the stress level has been taken into account in different ways, e.g.

(i) The secant friction angle ϕ_s' was supposed to be a function of a mean stress level, de Beer (1965) which by Meyerhof (1948) was suggested to be

$$\sigma_{g,M}' = 0.1 \frac{Q}{A} \tag{12}$$

(ii) The secant friction angle ϕ_s' was supposed to vary with the mean normal stress σ_m'. The shear stress and normal stress were calculated along the slip lines (Graham and Stuart (1971), Graham and Pollock (1972)).

In this paper the influence of stress level is expressed in eq (5), which can be used directly to calculate the combined q- and c'-term in the bearing capacity of a strip footing in eq (3).

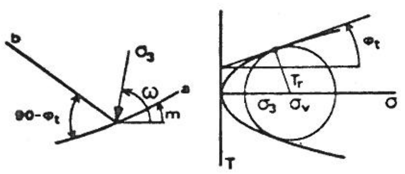

Fig. 5: Definition of stress and strength parameters.

The stress variables σ_v', τ_r (Fig. 5) along the a-slip line can be expressed in differential form:

$$\frac{\partial \sigma_v'}{\partial s_a} \cos\phi_t + 2\tau_r \frac{\partial \theta}{\partial s_a} = 0 \tag{13}$$

or

$$\frac{\partial \sigma_3'}{\partial \theta} + 2\tau_r \tan(45 - \frac{\phi_t'}{2}) = 0 \tag{14}$$

$\partial\theta = \partial m + \partial\phi_t'$. m is defined in Fig. 5. τ_r from eq (5) with $(\phi_a, c_a) = (\phi_d', c_d')$ inserted into eq (14) yields a differential equation from which σ_3' can be determined as a function of ω. ω is defined in Fig. 5. The boundary condition is $\sigma_3' = q$ for $\omega = 90^\circ$. The bearing capacity is then

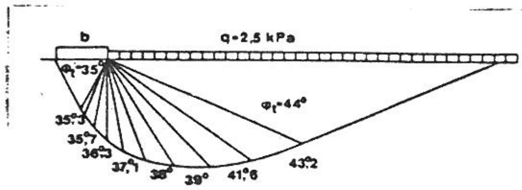

Fig. 6: Variation of ϕ_t' in the rupture figur.

$$b = \sigma_3'^{0} + 2\tau_r^0$$

where $\sigma_3'^{0}$ and τ_r^0 corresponds to $\omega = 0$.

The limit slip line and the radial slip lines are shown in Fig. 6 for a soil with $\phi_a = 34^\circ$, $c_a = 12$ kPa and $m = 0.25$.

ANALYSIS OF PLATE TESTS

Two model tests on Blokhus sand are shown in Fig. 7. Even after failure the bearing capacity b increases because the effective overburden pressure at base level and the depth factor d_q increase.

It is assumed that the depth factor is

$$d_q = 1 + 0.35\,\delta/B \tag{16}$$

The bearing capacity b for $\delta = 0$ is determined by eliminating the increase in d_q and the unit weight pressure $\gamma\delta$ as indicated by straight lines in Fig. 7.

All plate tests have been treated in that way and the bearing capacity b versus the surface load q is shown in Fig. 7-10. The scape factors s_q and s_c are assumed to be

$$s_q = s_c = 1 + 0.2\,B/L \tag{17}$$

Most of the tests were carried out with B/L = 1/3 to reduce the influence of the shape factors.

The bearing capacity is calculated as mentioned in the previous section with reduced asymptotic strength parameters $\phi_{d,a}$ and $c_{d,a}$ where $\phi_{d,a}$ is determined by fitting the curve to the test

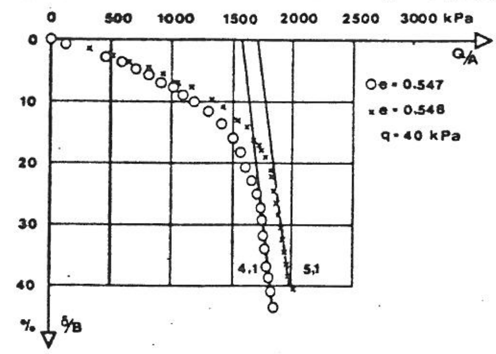

Fig. 7: Definition of bearing capacity in model tests.

result and $c_{d,a}$ from $c_{d,a} \cdot \cot\phi_{d,a} = \bar{c_a} \cdot \cot\phi_a$, see Fig. 8-11.

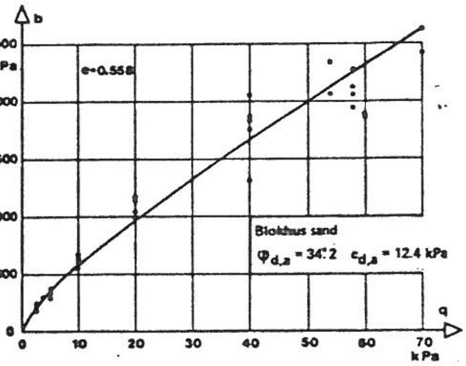

Fig. 8: 45 model tests (B x L = 3.5 x 10.5 cm) on Blokhus sand.

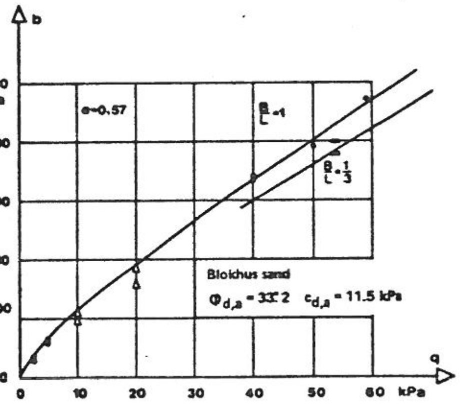

Fig. 9: 14 model tests (B x L = 5 x 5 and 3.5 x 10.5 cm) on Blokhus sand.

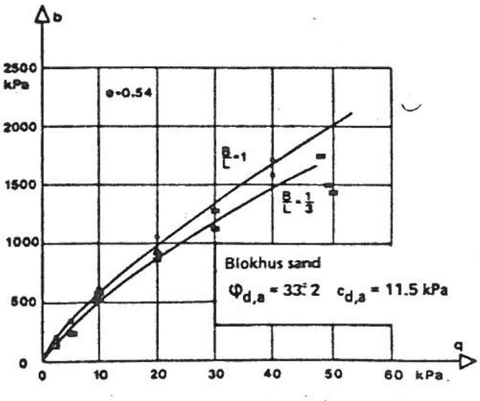

Fig. 10: 25 model tests (B x L = 5 x 5 and 3.5 x 10.5 cm) on Blokhus sand.

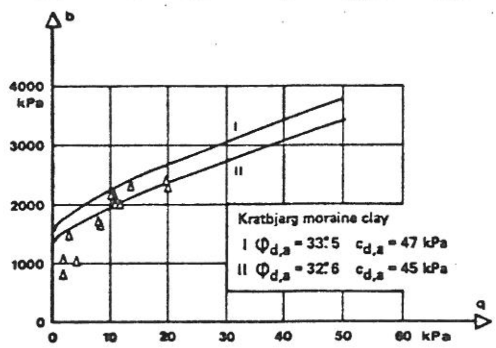

Fig. 11: 13 drained field tests on moraine clay.

From this it can be concluded, that

a) the introduction of m in the modified Coulomb failure criterion, eq (5) describes the influence of the stress level satisfactorily,

b) the absence of an associated flow rule for friction material is very important and reduces the plane strength parametres considerably, as shown in Table II,

c) the shape factors seem to be as proposed in eq (17). The shape factors are much smaller than usually determined in plate tests as mentioned by Weiss (1973) and de Beer (1970).

There can be at least two explanations

(i) model tests with relatively small surface load have a γ-term which equals or exceeds the q- and c-terms. The model laws are not fulfilled for such tests.

(ii) for small values of q the superposition factor μ_q can be defined by

$$b = \frac{1}{2}\gamma B N_\gamma s_\gamma + \mu_q s_q q N_q$$

where $\mu_q \to 2$ for $q \to 0$ (Bent Hansen 1975). Even in full scale tests $\mu_q s_q > 2$ could therefore be expected.

But it is not possible to explain the differens in shape factor between the tests mentioned in this paper and the tests of de Beer (1970).

Table II

			Blokhus sand			Kratbjerg moraine clay
			Fig. 8	Fig. 9	Fig. 10	
Void ratio	e		0.56	0.57	0.54	0.32
Triaxial	ϕ_a		34°2	33°7	35°	32°6
compression	c_a	kPa	12.4	12.0	13	45
state	m		0.25	0.25	0.25	0.8
Plane state	$\phi_{pl,a}$		37°6	37°	38°5	35°9
(assumed)	$c_{pl,a}$	kPa	14	13.6	14.8	51
Reduced parameters	$\phi_{d,a}$		34°2	33°2	33°2	32°6 33°5
(determined by model tests)	$c_{d,a}$	kPa	12.4	11.5	11.5	45 47

CONCLUSION

The effective strength parametres c' and ϕ' as measured in triaxial tests depend on the stress level. This dependence can be included into an extended Coulomb failure criterion eq (5) introducing a third parameter m. The plane strength parametres c'_{pl} and ϕ'_{pl} should in principle be used in calculation of the bearing capacity of footings.

Since the angle of dilatation ν is smaller than the angle of effective friction ϕ' no associated flow rule exists and a statically determined solution will be an upper bound solution. The statically determined solution can nevertheless be applied, provided reduced values of the strength parameters are used (Table II). This is demonstrated through the calculation of the bearing capacity of model footing in Fig. 8-11, where the influence of the stress level has been taken into account by using the extended Coulomb failure criterion eq (5).

It should be noted that the model tests were carried out with all dominating surface load in order to fulfil the model laws. The grain size however, was not reduced and thus did not conform to the model laws. But it is known to be of minor importance for bearing capacity. Because of the dominating surface load the results do not apply to the γ-term in eq (1).

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